CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the October/November 2013 series

4037 ADDITIONAL MATHEMATICS

4037/22 Paper 2, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

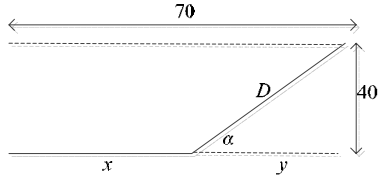
- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

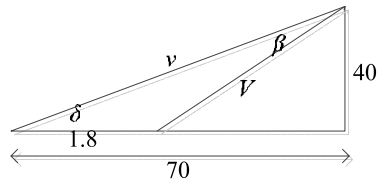
- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

1	$(x+6)(x-1)$ Critical values –6 and 1 $-6 < x < 1$	**M1** **A1** **A1** **[3]**	Attempt to solve a three term quadratic Allow $x > -6$ **AND** $x < 1$ but not **OR** or a comma. Mark final answer.
2	$\left(4\sqrt{5}-2\right)^2 = 80-16\sqrt{5}+4$ Multiply top and bottom by $\sqrt{5}+1$ $17\sqrt{5}+1$ **OR** $\left(4\sqrt{5}-2\right)^2 = 80-16\sqrt{5}+4$ $\left(\sqrt{5}-1\right)\left(p\sqrt{5}+q\right) = 5p-q+\sqrt{5}(q-p)$ Leading to $5p-q=84, q-p=-16$ $p=17 \quad q=1$	**M1** **M1** **A1 A1** **[4]** **M1** **M1** **A1 A1**	Attempt to expand, allow one error, must be in the form $a+b\sqrt{5}$. Must be attempt to expand top and bottom. Allow A1 for $\frac{68\sqrt{5}+4}{c}$ Must get to a pair of simultaneous equations for this mark
3 **(i)**	$\frac{\mathrm{d}y}{\mathrm{d}k} = k\left(\frac{1}{4}x-5\right)^7$ $k=2$	**M1** **A1** **[2]**	
(ii)	Use $\partial y = \frac{\mathrm{d}y}{\mathrm{d}x} \times \partial x$ with $x=12$ and $\partial x = p$ $-256p$	**M1** **A1**√ **[2]**	√ on k needs both M marks √ only for $-128kp$ and must be evaluated
4 **(i)**	10	**B1** **[1]**	
(ii)	–5	**B1** **[1]**	Not $\log_p 1 - 5$
(iii)	$\log_p XY = \log_p X + \log_p Y = 7$ $\frac{1}{7}$	**B1** **B1**√ **[2]**	Or $\log_{XY} p = \frac{1}{\log_p XY}$ Do not allow just $\log_p X + \log_p Y = 7$ √ on $\frac{1}{\log_p XY}$

5	$x-4y=5$ oe $2x+2y=5$ oe Solve their linear simultaneous equations $x=3$ or $y=-0.5$ **OR** from log $0.602x-2.408y=3.01$ $0.954x+0.954y=2.386$ **OR** from ln $1.386x-5.545y=6.931$ $2.197x+2.197y=5.493$ Final M1A1A1√ follows as before	**B1** **B1** **M1** **A1,A1√** **[5]** **B1** **B1** **B1** **B1**	Each in two variables and not quadratic as far as x = … or y = …
6 **(a)** **(i)**	–8 or 20 $-160\left(x^3\right)$ isw	**B1** **B1** **[2]**	± 40 implies $\pm 2\times 20$ or +160 hence B1 OK if seen in expansion
(ii)	$60\left(x^2\right)$ (i) $+\frac{1}{2}$ (their 60) $-130\left(x^3\right)$	**B1** **M1** **A1** **[3]**	Can be implied
(b)	$16x^2+32x+24+\frac{8}{x}+\frac{1}{x^2}$ oe	**B3,2,1,0** **[3]**	Terms must be evaluated (allow $24x^0$) B2 for 4 terms correct. B1 for 2 or 3 terms correct. ISW once expansion is seen.
7 **(i)**	$l=\frac{3500}{x^2}$ $L=3\times 4x+2x+2l$ Substitute for l and correctly reach $L=14x+\frac{7000}{x^2}$	**B1** **B1** **DB1ag** **[3]**	allow $lx^2=3500$ RHS 3 terms e.g. $12x+2x+2\left(\frac{3500}{x^2}\right)$ or better Dependent on both previous B marks
(ii)	$\frac{dL}{dx}=14-\frac{14000}{x^3}$ Equate $\frac{dL}{dx}$ to 0 and solve $x=10$ $L=210$ $\frac{d^2y}{dx^2}=\frac{42000}{x^4}$ and minimum stated	**M1A1** **DM1** **A1** **B1** **[5]**	M1 either power reduced by one A1 both terms correct Must get x^n = Both values Or use of gradient either side of turning point.

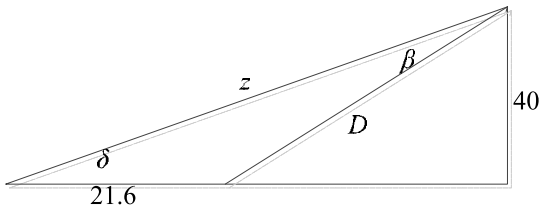
8	**(i)**	x^2	**B1** **[1]**	Implied by axes or values in a table. May be seen in (ii)
	(ii)	Plot $\frac{y}{x}$ against x^2 with linear scales		Must be linear scales
		x^2 4 16 36 64	**B1**	At least 3 correct points plotted and no incorrect points
		$\frac{y}{x}$ 4.8 9.6 17.5 29	**B1** **[2]**	Line must be ruled and through at least 2 correct points
	(iii)	Finds gradient (0.4) $a = 0.4 \pm 0.02$ $b = 3.2 \pm 0.4$	**M1** **A1** **B1** **[3]**	Condone use of correct values from table/graph to find gradient and /or equation. Values read from graph must be correct.
	(iv)	Read $\frac{y}{x} = 12.5$	**M1**	Obtaining $(x^2) = 22$ to 24 from graph
		or substitute in formula		As far as x^2 = +ve constant
		4.8	**A1** **[2]**	4.7 to 4.9 ignore –4.8 or 0
9		Method A Takes components $12v\sin\alpha = 40$ $12(v\cos\alpha + 1.8) = 70$ $12v\cos\alpha = 48.4$ Solve for v or α $\alpha = 39.6$ $v = 5.23$	**M1** **A1** **A1** **M1A1** **DM1** **A1** **A1** **[8]**	Allow 0.691 radians
		Method B  $x = 1.8 \times 12 = 21.6$ $y = 70 - 21.6 = 48.4$ $D^2 = 40^2 + 48.4^2 (= 3942.56)$ $D = 62.8$ $V = \frac{D}{12}$ $V = 5.23$ $\tan\alpha = \frac{40}{48.4}$ $\alpha = 39.6°$	**B1** **B1** **M1** **A1** **DM1** **A1** **M1** **A1** **[8]**	5.23 or better Allow 0.691 radians

Method C



$z=\sqrt{40^2+70^2}\,(=80.6)$	**B1**	
$v=\dfrac{\sqrt{40^2+70^2}}{12}\,(=6.72)$	**B1**	
$\tan\delta=\dfrac{4}{7}\rightarrow(\delta=29.74)$ oe	**B1**	Or $\tan(90-\delta)=\dfrac{7}{4}$
$V^2=1.8^2+6.72^2-2\times1.8\times6.72\cos 29.74$	**M1**	
$V=5.23$	**A1**	
$\dfrac{\sin\beta}{1}.8=\dfrac{\sin 29.74}{5}.23$	**M1**	
$\beta=9.8(3)$ or $9.8(2)$	**A1**	Allow 0.172 radians
$\alpha=29.74+\beta=39.6$	**A1**	Allow 0.691 radians
	[8]	

Method D



	B1	
$z=\sqrt{40^2+70^2}\,(=80.6)$	**B1**	
$x=1.8\times12=21.6$	**B1**	
$\tan\delta=\dfrac{4}{7}\rightarrow(\delta=29.74)$ oe	**M1**	
$D^2=21.6^2+80.6^2-2.21.6.80.6\cos 29.74$	**A1**	This method has extra steps so note at this point the M mark is for an equation in D but the A mark is for a value of V.
$V=(62.8/12)=5.23$		
$\dfrac{\sin\beta}{21}.6=\dfrac{\sin 29.74}{62}.8$	**M1**	
$\beta=9.8(3)$ or $9.8(2)$	**A1**	Allow 0.172 radians
$\alpha=29.74+\beta=39.6$	**A1**	Allow 0.691 radians
	[8]	

10	**(i)**	$AB^2 = 12^2 + 12^2 - 2\times12\times12\times\cos1.4$	**M1**	$AB = 2\times12\sin0.7$
		15.4 to 15.5	**A1**	May be implied
		$\theta = 2\pi - 1.4 (= 4.88)$	**B1**	May be implied
		Use $s = r\theta (= 58.6)$	**M1**	12×4.9 or better oe
		74.1	**A1**	
			[5]	
	(ii)	(Sector) $\frac{1}{2}\times12^2\times(2\pi - 1.4)(= 352)$ or	**M1**	May be implied .
		$\pi\times12^2 - \frac{1}{2}\times12^2\times1.4$		
		(Triangle) $= \frac{1}{2}\times12\times12\times\sin1.4 (= 70.9 \text{ or } 71)$	**M1**	
		Area of **major** sector + Area of triangle	**M1**	May be implied
		422 or 423	**A1**	
			[4]	
11	**(i)**	$\frac{dy}{dx} = \frac{1}{3}e^{\frac{1}{3}x}$	**B1**	
		$m = \frac{1}{3}e^3$	**M1**	For insertion of $x = 9$ into their $\frac{dy}{dx}$. 6.7 or better if correct.
		$y - e^3 = \frac{1}{3}e^3(x - 9)$	**DM1**	Using their evaluated m to find eqn $y = 6.7x - 40.2$ or better if correct.
		At Q $y = 0$, $x = 6$	**A1**	Accept value that rounds to 6.0 to 2sf
			[4]	
	(ii)	Area triangle $1.5e^3$ or 30.1	**B1**	
		$\int e^{\frac{1}{3}x}dx = 3e^{\frac{1}{3}x}$ oe	**B1**	
		Uses limits of 0 and 9 in integrated function.	**M1**	± must see both values inserted if incorrect answer
		$3e^3 - 3$ or 57.3	**A1**	
		Area under curve subtract area of triangle	**M1**	
		$1.5e^3 - 3$ or 27.1	**A1**	Condone 27.2 if obtained from 57.3 – 30.1.
			[6]	

12	**(a)**	$\text{cosec}x = \frac{1}{\sin x}$ inserted into equation	**B1**	
		$\tan x = -\frac{2}{7}$	**DB1**	
		164.1	**B1**	One correct value.
		344.1	**B1**√	√ on $180 + (164.1)$ Must come from tan$x$ =
			[4]	Condone164 and 344 Deduct 1 mark for extras in range
	(b)	$(2y - 1) = 0.79..$or $2.34\ldots$	**B1**	Allow 0.8 , 2.3 or 45.6°
		Find y using radians	**M1**	Add 1 then divide by 2 on a correct angle
		0.898 (or 0.9 or 0.90)	**A1**	One correct value
		1.67, 4.04 and 4.81(45)	**A1**	Another correct value
			A1	Final two values
			[5]	Deduct 1 mark for extras in range